Lecture 27. Arrow’s impossibility result
Outline

1) Arrow’s impossibility result
2) Universal domain
3) Collective rationality
4) Weak pareto condition
5) Independence of irrelevant alternatives
6) Deliberative democracy
The theorem

- Arrow’s impossibility result states that for a group of at least three individuals who have a choice between at least three alternatives, there is no social choice function that simultaneously fulfills the conditions of collective rationality, universal domain, weak Pareto, independence of irrelevant alternatives, and non-dictatorship.

- Let us recall the definitions of these **five conditions**
Arrow’s impossibility result

The conditions

1) **Collective rationality**: The social ranking $\succeq$ of the alternatives is transitive and complete for any given complete and transitive individual preference profiles.

2) **Universal domain**: There is no restriction on the way in which individuals may rank the alternatives, except that their ranking is consistent. In other words, no matter how the votes come in, the social choice function specifies a unique social preference ordering.

3) **Weak Pareto condition**: Society’s ranking is required to be positively responsive to the individual rankings in the following sense: if, for all (i), $x >_i y$, then $x >_s y$. In other words, if an option is individually preferred by everyone, then it also socially preferred.
The conditions (continued)

- **4) Independence of irrelevant alternatives**: The way society ranks two alternatives $x$ and $y$ should only depend on the individuals' rankings of $x$ and $y$, and not on their rankings of other (irrelevant) alternatives.

  This means that the relative social rankings of $x$ and $y$ should not change when outcomes other than $x$ and $y$ are eliminated from the option set, and the social choice function is reapplied to the modified option set.

- **5) Non-dictatorship**: There is no individual such that society always ranks alternatives in the same way as that individual. Stated differently, no single individual shall dictate the social choice.
Implications

- Arrow's theorem has a large number of potential applications. Arrow's own target was the New Welfare Economics of Samuelson, Bergson and Little, and their notion of a social welfare function.

- Arrow and many others took his result to show that such functions do not exist, and thus welfare economics has no object of study.

- William Riker and others have seen in Arrow's impossibility result a direct challenge to democratic theory, at least to the kind that appeals to some notion of the 'general will' (Rousseau).

- However, before one can draw such pessimistic conclusions, one needs to make sure that Arrow's conditions are plausible, and that we want all of them to be fulfilled by a social choice function.

- In the last class, we already discussed the non-dictatorship condition. So, let us focus on the other four assumptions in the following...
Single-peaked preferences

- As our discussion of Condorcet’s voting paradox has shown, if we restrict the domain to single-peaked preferences, then we do not face an impossibility result.
- Stated differently, if the domain is restricted to single-peaked preferences, then there are non-dictatorial social choice functions, including majority voting.

Interdependence of preferences

- The universal domain condition entails the assumption that the preferences of the members of society are independent from each other in the sense that they are given and stable for the voting procedure.
- However, the preferences of the members of a society are typically interdependent, because the group members influence each other in their decisions and they have often shared values.
- More precisely, it may be constitutive for being a group that the group members have interdependent preferences.
Choices and true preferences

- In social choice theory, preferences are identified by the individuals’ actual choices (revealed preferences).
- Choices, however, do not necessarily express the individuals’ ‘true’ preferences. Take the example of adaptive preferences, counter-adaptive preferences (the grass is always greener on the other side of the fence), and strategic preferences.
- Should those preferences be taken into account for the aggregation process although they do not express the individuals’ true preferences?

Immoral preferences

- Harsanyi argues that we should not respect anti-social preferences for social choices, such as preferences based on envy and malice, for instance.
- If we exclude anti-social preferences, and other socially ‘unwelcome’ preferences as an input for the aggregation procedure, then we may be able to avoid Arrow’s impossibility result.
Procedures versus outcomes

- Buchanan ("Social Choice, Democracy, and Free Markets", 1954) argues that Arrow’s impossibility result is not surprising.
- Why should we expect that aggregative decision-making exhibits the same rationality as individual decision-making?
- Buchanan argues that attributing rationality to groups implies that these ‘artificial’ entities have some kind of existence independent of the individuals who make them up, which is not the case.

- **Public choice theory** has largely taken Buchanan’s point on board by rejecting consequence-based evaluations of social decisions in favor of **procedural criteria**. Priority is given to the ‘right’ procedures over ‘good’ outcomes.
- The **rightness** of certain procedures is more important than the **goodness** of the outcomes.
Procedures versus outcomes (continued)

- In particular, Buchanan defends the **market mechanism** as a means of making ‘consistent choices’

- The idea is **not** that the market calls upon individuals to make a **collective decision**. Instead, the market offers a **procedure** that leads to consistent choices based on individual decision-making

- To avoid misunderstanding, the market does not allow to derive a ‘social welfare function’ from the individual rankings that are implicit in the market mechanism, which would make it possible to predict the social result in advance

- Instead, one has to **wait and see** what the market produces in order to identify the socially optimal outcome
Concerns

- Buchanan’s criticism does not apply to social welfare rankings, because social welfare rankings need not involve imputation of agency to society.
- The rankings could be constructed by an individual, for instance, on the basis of her judgment of what would be the best outcome for society.
- Further, transitivity and completeness can be regarded as desiderata for a social decision mechanism without dressing them up as properties of an agency based rationality.
- In short, if social preferences are intransitive, it is simply impossible for policy-makers to make a decision, because there is always an option that is better.
Weak Pareto condition

- The Pareto condition is questionable when unanimous preferences rest upon disagreements about the reasons supporting these preferences.
- For instance, I might prefer policy A, because I believe that it will lead to consequence c, which I prefer to consequence c*. You prefer policy A, because you believe that it will have consequence c*, which you prefer to consequence c.
- Our unanimous preference for A is spurious, because it arises from real disagreements in two dimensions cancelling themselves out.
- This, of course, points to the deeper issue of whether preferences based on false beliefs should be respected at all. But applying the Pareto condition to those preferences seems to be particularly worrisome.
- Further, as we have seen from our discussion of Sen’s liberal paradox, although the Pareto condition is often thought to be an expression of individual liberty, it can lead to illiberal consequences.
Independence of irrelevant alternatives

Independence axiom

- The IIA condition combines two principles:
  - a) **Independence**: Only information pertaining the given option set is used in determining society's ranking of the possible options
  - b) **Ordinalism**: The only pertinent information is the individuals' rankings of the alternatives, and not their preference strength

- Jointly, these two principles disallow **comparative judgments** that seem relevant in many situations
Independence axiom (continued)

- Consider, for instance, the following case where person A and B differ on the ordering of the set \{a, z\}: (A): \(a > b > \ldots > y > z\); (B): \(b > \ldots > z > a > \ldots > y\)

- The independence of irrelevant alternatives axiom requires one to reduce the situation to the following information: person A prefers \(a > z\), and person B prefers \(z > a\)

- The fact that A prefers option (a) ‘much more’ over (z) than B prefers (z) over (a), measured by the number of options between (a) and (z), is not taken into account, but only the overall rankings

- Dropping either independence or ordinalism will provide an escape from Arrow’s impossibility result

- If we drop independence, then we can make ordinal comparative judgments. If we drop ordinalism, then we allow for all kinds of alternative information
Independence axiom (continued)

- For instance, by using information about the relative strength of individual preferences, utilitarian and other welfarist rules become possible.

- Then, however, we need to deal with the problem of interpersonal comparisons of utility, as discussed in week 9.

- According to Harsanyi, this problem is not too worrisome. Harsanyi argues that although full comparability of utilities is not possible, human beings can make interpersonal comparisons of utility of some kind, because they share a common human nature, cultural background, etc.

- This idea is expressed by Harsanyi’s notion of extended preferences. In short, Sen agrees with Harsanyi that we can make interpersonal comparisons of utility to some extent. Sen even argues that we cannot avoid doing so with regard to issues such as poverty, famine, inequality, etc.
Sen’s broadening of the informational basis

- Specifically, Sen argues that the rejection of interpersonal comparisons of utility was mainly because the notion of utility was meant to represent mental states.
- However, if one uses resources as a basis for welfare comparisons, then this problem is less pertinent.
- If we broaden the informational basis of social choice theory this way, then Arrow’s impossibility result does not necessarily arise.
- As Sen (1982) puts it: “It is the imposed poverty of the utility information that dooms Arrow’s aggregation exercise to failure.”
- Further, Arrow’s impossibility result can help us to develop alternative frameworks to social choice, such as deliberative democracy.
Deliberative democracy is one of the ‘hot topics’ in current political science. The most prominent defenders are Cohen, Gutmann, and Dryzek.

Deliberative democracy particularly criticizes the **universal domain assumption**, which requires individual preferences to be independently formed and exogenously given for the choice procedure.

Deliberative democracy rejects this **static approach** of social choice theory by assuming preferences to be shaped in the political forum via a process of public deliberation.

The core assumption is that individual preferences may **change** in the public decision-making process.

One forerunner of this idea was Jürgen Habermas’ **discourse ethics**, as Jon Elster points out.

- Elster distinguishes between **three different types of political theory**: social choice theory, Habermas’ discourse ethics, and ideal democracy

- **1) Social choice theory**: Elster argues that social choice theory regards political processes as purely instrumental. Political results are seen as **compromises** between given and irreducibly conflicting private interests.

- The political forum is like the market and the act of voting like private selling and buying.

- **2) Consensus model**: This view of politics is represented by Habermas’ discourse ethics. Habermas regards **rational agreement** as the goal of politics, and not compromises. Further, politics is not private, but it requires public debate that is aimed at reaching a consensus among the citizens.
3) Participatory democracy: The ideal of participatory democracy was defended by Mill in particular. This view of politics follows the consensus model by regarding politics as non-private and non-instrumental. However, it is more idealistic by taking the goal of politics to be the transformation and education of citizens. Politics is seen as an end in itself (recall our discussion of Mill in week 12)

- Deliberative democracy falls into category 2), maybe with some elements of 3)
- Deliberative democracy regards politics as a process of mutual deliberation that leads, in the ideal case, to a rational consensus
- If an agreement can be reached among citizens on political issues, then no aggregation procedure is needed
- In this case, Arrow's impossibility result becomes irrelevant
- But how can a consensus be reached among the citizens of a society?
The idea of public rationality is that if individuals need to develop their political views in a public debate, in a discourse with their fellows, then they are pressed to think about what is good for the group as a whole rather than for themselves, because they need to justify their positions to their fellows.

Group debates force the members of society to abstract from their private interests and to take the interests of their fellows into account.

In order for the deliberation process to be successful, the public debate must take place under ideal conditions: equal say, respect, openness for reasons (no dogmatism), freedom of speech, etc.

If these ideal conditions do not hold, we return to the ‘bad’ old world of bargaining between antecedently given preferences.
Deliberation versus aggregation?

- There is some debate on whether public deliberation could replace voting procedures entirely (unanimity)?
- This, however, seems to be unrealistic, because it is unlikely that a full consensus will be reached on all public issues in a pluralistic society.
- Moreover, the process of deliberation may also have the opposite effect: it may make the citizens aware of their different, irreconcilable positions.
- As such, it seems more realistic to argue for a **two-step procedure**: first deliberation, and then voting.
- The advantage of such a two-step procedure is that it ensures that the preferences which serve as ‘input’ for the voting procedure are **restricted in certain ways**.
Deliberation versus aggregation? (continued)

- The reasons for such restriction are:
  - 1) Preferences based on false beliefs can be identified and excluded
  - 2) Narrowly self-regarded preferences can be eliminated
  - 3) Strategic preferences can be better identified, and thus ruled out
  - 4) Completely a-social preferences may not be uttered in public at all
  - 5) Discussion may foster social cohesion (group interest)

As such, the process of deliberation may lead to **single-peaked preferences**, because single-peakedness shows that the voters understand the public choice in the same way, even though they adopt different positions on the political spectrum.
Deliberation versus aggregation? (continued)

- Further, deliberation can help to render the decision problem **one-dimensional**, because public deliberation will show whether the issue to be decided involves too many different criteria that are differently weighted by the voters. As a result, one can try to split up the issue in order to make it one-dimensional

In sum

- The process of deliberation can help to clarify the nature of the individual preferences and the nature of the issue to be decided
- As a consequence, the decision problem can be formulated in a less ambiguous way, and the most adequate decision-making rule can be identified for the particular issue to be decided
- Moreover, the process of deliberation may harmonize the group members’ preferences so that the impossibility result in the aggregation process can be avoided